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NEIGHBOR DIFFUSION IN
MONTEREY BAY

by

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United States Naval Postgraduate School



THESIS

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October 1969

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Neighbor Diffusion in Monterey Bay

by

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OCEANOGRAPHY

from the

NAVAL POSTGRADUATE SCHOOL
October 1969

ABSTRACT

The applicability of Richardson's "four-thirds law" to turbulent horizontal diffusion, the dependence of diffusion on the weight of the diffusers, and the effects of varying the rejection level of the data were investigated. Diffusers of identical size and shape but weighing 13 or 15 pounds were photographed from a Navy US2-A aircraft. The data were collected in Monterey Bay on scales from 25 meters to 200 meters, with water depths ranging from 220 to 270 feet.

The results indicate a nearly constant value of k (the constant in Richardson's "four-thirds law", ie. $F(t) = kt^{4/3}$) for Monterey Bay, although in nearly all cases the slope of the best-fit line was greater than predicted by Richardson (1926). The weight effect remains unsettled. Varying the rejection criterion has definite effects on k and on the slope of the best-fit line.

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ACKNOWLEDGEMENTS

The author wishes to express his appreciation to Assistant Professor Theodore Green, III for his invaluable guidance and assistance in this study.

A very special thanks to Sandra Johnson for her willing assistance and encouragement throughout the entire study.

I. INTRODUCTION

A. HORIZONTAL TURBULENT DIFFUSION AND RICHARDSON'S "4/3" LAW

Richardson (1926), after analysis of atmospheric experimental data, noted that in molecular diffusion the motion of each molecule is independent of its immediate neighbors, whereas, in turbulent diffusion it is not. Then the classical Fickian equation $\frac{\partial v}{\partial t} = K \frac{\partial^2 v}{\partial x^2}$, where v is the concentration of a diffusing substance, x is position, t is time, and K is the diffusivity (a measure of the rate of diffusion), cannot be applied to turbulent diffusion, for it predicts a constant rate of diffusion with x . Stommel (1949) pointed out that using the Fickian equation the probability of two particles initially a distance of l_0 apart being a distance of l_1 apart at some later time, would be

$$P(l_0, l_1) = \frac{1}{2\sqrt{\pi Kt}} \exp\left(-\frac{(l_1 - l_0)^2}{4Kt}\right).$$

Therefore, the Fickian equation leads to the result that the probability of a pair of particles a distance of l_0 apart being a distance of l_1 apart after an interval of time t depends upon $(l_1 - l_0)^2$ only, and not upon either l_0 or l_1 . This is clearly in direct contradiction to the atmospheric observations. There seems to be no way in which the Fickian equation can be modified to overcome this discrepancy.

On the basis of atmospheric diffusion experiments, Richardson (1926) postulated the equation

$$\frac{\partial q}{\partial t} = \frac{\partial}{\partial l} \left[F(l) \frac{\partial q}{\partial l} \right].$$

Here, l is the distance between particles and is called the "neighbor separation". The number of particles which have neighbors with neighbor

separations of between l and $l + dl$ is $q(l)dl$. The quantity $q(l)$ may then be called "neighbor concentration". The term $F(l)$ is the "neighbor diffusivity" and is analogous to the diffusivity (K) in the Fickian equation.

Richardson empirically determined that neighbor diffusivity was related to neighbor separation by $F(l) = kl^{4/3}$ where k is a constant. This is commonly referred to as the "four-thirds law". Stommel (1949), on the basis of data obtained from aerial photography, suggested that the four-thirds law was also applicable to oceanic diffusion. Then, assuming the turbulence causing the diffusion is within the inertial subrange, it can be shown that $k = CE^{1/3}$, where C is a universal dimensionless constant and E is the turbulent energy dissipation per unit mass.

Many individual studies have been conducted to test Richardson's four-thirds law. While general agreement exists as to the validity of the law, the data have not been sufficient to actually confirm it. Attempts have been made to determine k , but very little work has been done on evaluating C . The reported values of k range from 0.003 to $0.09 \text{ cm}^{2/3} \text{ sec}^{-1}$. (These values were found by passing a best-fit line of $4/3$ slope through the data points plotted on logarithmic paper.)

B. A METHOD FOR THE DETERMINATION OF THE NEIGHBOR DIFFUSIVITY AS A FUNCTION OF THE NEIGHBOR SEPARATION

Consider two particles initially separated by a distance l_0 at time t_0 . At a later time t_1 the separation of particles is l_1 . If the time interval $t_1 - t_0$ is chosen so that the ratio $\frac{l_1 - l_0}{l_0}$ is small,

Richardson's equation can be written as

$$\frac{\partial q}{\partial t} = F(t_0) \frac{\partial^2 q}{\partial t^2}$$

which has the solution

$$q(t_1) = \frac{\text{const.}}{\sqrt{t}} \exp \left[- \frac{(t_1 - t_0)^2}{4t E(t_0)} \right].$$

Thus the neighbor separation has a Gaussian distribution. The standard deviation of t_1 from the mean t_0 is $\sigma = \sqrt{2\Delta t F(t_0)}$. Thus $F(t_0) = \frac{\sigma^2}{2\Delta t}$. The best estimate of σ results in $F(\bar{t}) = \frac{\sum_{i=1}^N (\Delta t_i)^2}{(N-1)2\Delta t}$ where \bar{t} is an average of t_0 and t_1 (since all pairs of particles would not have the same initial separations), and N is the number of pairs averaged over.

C. OBJECTIVES OF THE STUDY

This study was conducted with three main objectives in mind. The first was to test the applicability of Richardson's four-thirds law to turbulent horizontal diffusion in the ocean under various field conditions. An investigation of the dependence of diffusion on the weight of the particles was the second objective. This was studied by comparing the diffusion of 13 pound and 15 pound plywood boards. (Earlier work had lead to the belief that even such a slight difference might be important.) The third objective was to study the effects of varying the rejection level of the data. That is, in most of the previous work the data were considered acceptable if the relative change in separation $R = \frac{2\Delta t}{t_0 + t_1}$ was on the order of 0.10. By giving R various upper limits it was felt that the data would be more useful and the effects, if any, of the different limits could be determined. This was accomplished by

calculating the neighbor diffusivity and the neighbor separation for all pairs with a relative change in separation of less than 0.05, less than 0.10, and less than 0.15.

II. EQUIPMENT AND PROCEDURES

A. TECHNIQUES USED IN COLLECTION OF DATA

The data used in this study were collected by Champion (1968). The particles were 4' x 4' x 1/8" plywood sheets, which were treated with sealer and marked for easy identification. The weights of the sheets were altered with the use of lead strips to within 0.1 pound of the prescribed weights of 13 and 15 pounds. The 13 pound sheets (A) were marked with a large red dot on a white background. The 15 pound sheets (B) were marked with two red horizontal stripes on a white background.

The sheets were photographed with a T-11 Fairchild aircraft mapping camera installed in the rear section of a Navy US2-A aircraft (Philipps, 1968). The camera was equipped with a 6-inch f:6.3 (Bausch and Lomb) class T metrogon lens that produced 9-1/2" x 10-1/4" negatives. The altitude of the camera to the nearest foot and time to the nearest second were automatically recorded on each negative.

The data were collected about 2300 yards offshore in Monterey Bay, with water depths ranging from 220 feet to 270 feet. The approximate initial location of the patterns was 36°-38.4'N and 121°-53.0'W. Circular patterns of about 280 feet in diameter were used. The time between each pass of the aircraft over the pattern was approximately two minutes. During each pass a series of three to six photos were taken to improve the possibility of getting the entire pattern into at least one photo for that pass. If a specific sheet was missing from a certain pass, it was located in the next pass and the analysis

continued. The final shape of the patterns for both runs was approximately elliptical with the major axis perpendicular to the direction of the mean current.

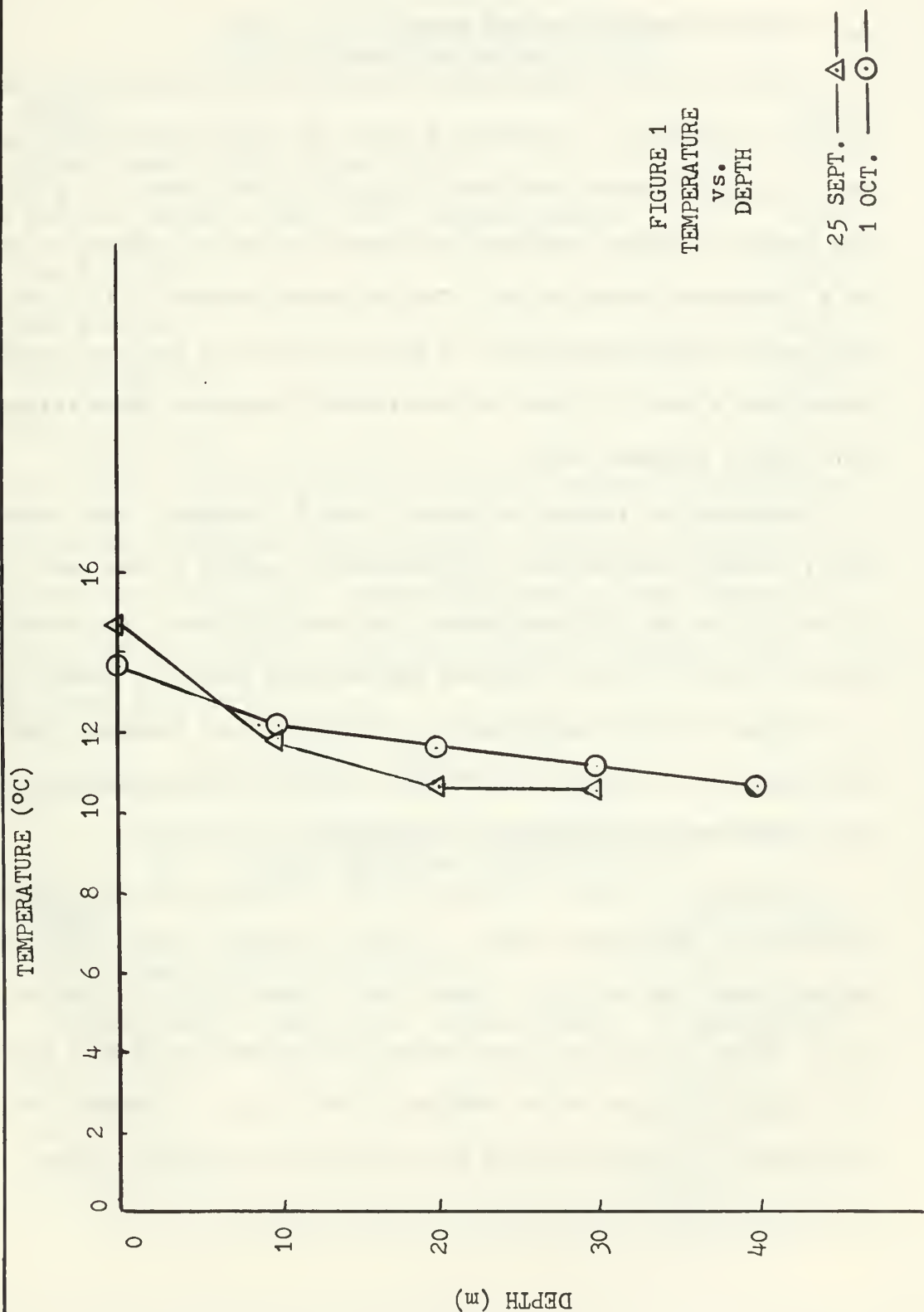
III. DATA ANALYSIS

Appendix A contains the pertinent data for the two flights used in this study. Data were analyzed from 24 September 1968 (Run I) and 1 October 1968 (Run II). The temperature structure of the upper layer on these days is shown in Figure 1. The negatives were used as transparencies and projected with a Travel-Graph overhead projector on an x-y grid. The grid was a sheet of 5' x 5' plywood painted white, with coordinates marked by telephone cable wire spaced at 6-cm. intervals. After the best transparencies were selected for each pass, each sheet was properly located and marked. The transparencies were then projected on the x-y grid. (The enlargement factor was 5.5.) The x-y coordinates of each sheet were determined, and a digital computer was used to calculate the initial separations (t_0). After removal of all duplicate data cards, another program computed the mean separations ($\frac{t_0 + t_1}{2}$), the change in separation (Δt), the relative change in separation (R), and neighbor diffusivity $\left[\frac{(\Delta t)^2}{2\Delta t} \right]$ for all pairs. Distances on the negative were converted to actual distances using the relationship:

$$\frac{\text{IMAGE}}{\text{FOCAL LENGTH}} = \frac{\text{GROUND COVERED}}{\text{ALTITUDE}}.$$

Histograms of the mean separations were drawn using three rejection factors: (i) $R < 0.05$, (ii) $R < 0.10$, and (iii) $R < 0.15$. Class intervals of the mean separations were established and the average value (over the class interval) of the mean separation and neighbor diffusivity $\left[\frac{\sum (\Delta t)^2}{(N-1)2\Delta t} \right]$ were found using a desk calculator. Very few

values were added by changing the rejection factor from 0.10 to 0.15. Thus, only data with (i) $R < 0.05$ and (ii) $R < 0.10$ were considered.



IV. RESULTS AND CONCLUSIONS

A. CONSISTENT RESULTS OF THE DATA

A total of 6,399 observations were used, covering scales from 25 meters to 200 meters. Appendix B gives the results obtained from these data. Eighty percent confidence limits were calculated for $F(\bar{l})$ using the number of values averaged over minus one as the degrees of freedom of a chi-square distribution. The calculated values of $F(\bar{l})$ and \bar{l} were plotted on logarithmic paper. A best-fit line, in the least squares sense, and a best-fit line of four-thirds slope were drawn through the data points (Figures 2-9).

Comparing the results of sheets A and B from Run I with those from Run II clearly demonstrates the consistent results of the data collected on the two different days. The best-fit lines for sheets A nearly coincide (Figure 10), and the best-fit lines for sheets B, even though they are separated by a small vertical distance, have approximately the same slope (Figure 11). The wave conditions in the two runs were quite different (see below).

In nearly all cases the slope of the best-fit line was greater than predicted by Richardson (1926). Table I compares these results with Snyder (1967) and Philipps (1968), for a common rejection factor of 0.10. Three of the four slope values determined from Runs I and II were larger than the value obtained by Philipps. A somewhat larger difference in slopes is noted when compared with Snyder. These differences will be discussed in a later section.

TABLE I

SLOPE VALUES FOR THE BEST-FIT LINE
(FOR A REJECTION FACTOR OF 0.10)

| <u>SOURCE</u> | <u>SLOPE</u> |
|--|--------------|
| Snyder (March, 1967) Paper sheets | 1.46 |
| Philipps (April, 1968) 10 lb. plywood sheets | 1.612 |
| Run I 13 lbs. | 1.62 |
| Run I 15 lbs. | 1.27 |
| Run II 13 lbs. | 1.86 |
| Run II 15 lbs. | 1.71 |

The values of k determined by passing a best-fit line of four-thirds slope through the data are shown in Table II (using a common rejection factor of 0.10). The near equality of the k values for Monterey Bay determined by various investigators suggest that k varies little with time.

TABLE II

k VALUES FOR THE BEST-FIT LINE OF FOUR-THIRDS SLOPE
(FOR A REJECTION FACTOR OF 0.10)

| <u>SOURCE</u> | <u>k (cm^{2/3} sec⁻¹)</u> |
|--|---|
| Snyder (March, 1967) Paper sheets | 0.006 |
| Philipps (April, 1968) 10 lb. plywood sheets | 0.006 \pm 0.002 |
| Run I 13 lbs. | 0.004 |
| Run I 15 lbs. | 0.003 |
| Run II 13 lbs. | 0.005 |
| Run II 15 lbs. | 0.005 |

FIGURE 2
NEIGHBOR DIFFUSIVITY F
vs.
NEIGHBOR SEPARATION $\bar{\ell}$

(Run I, 13 lb. sheets, $R < 0.05$)

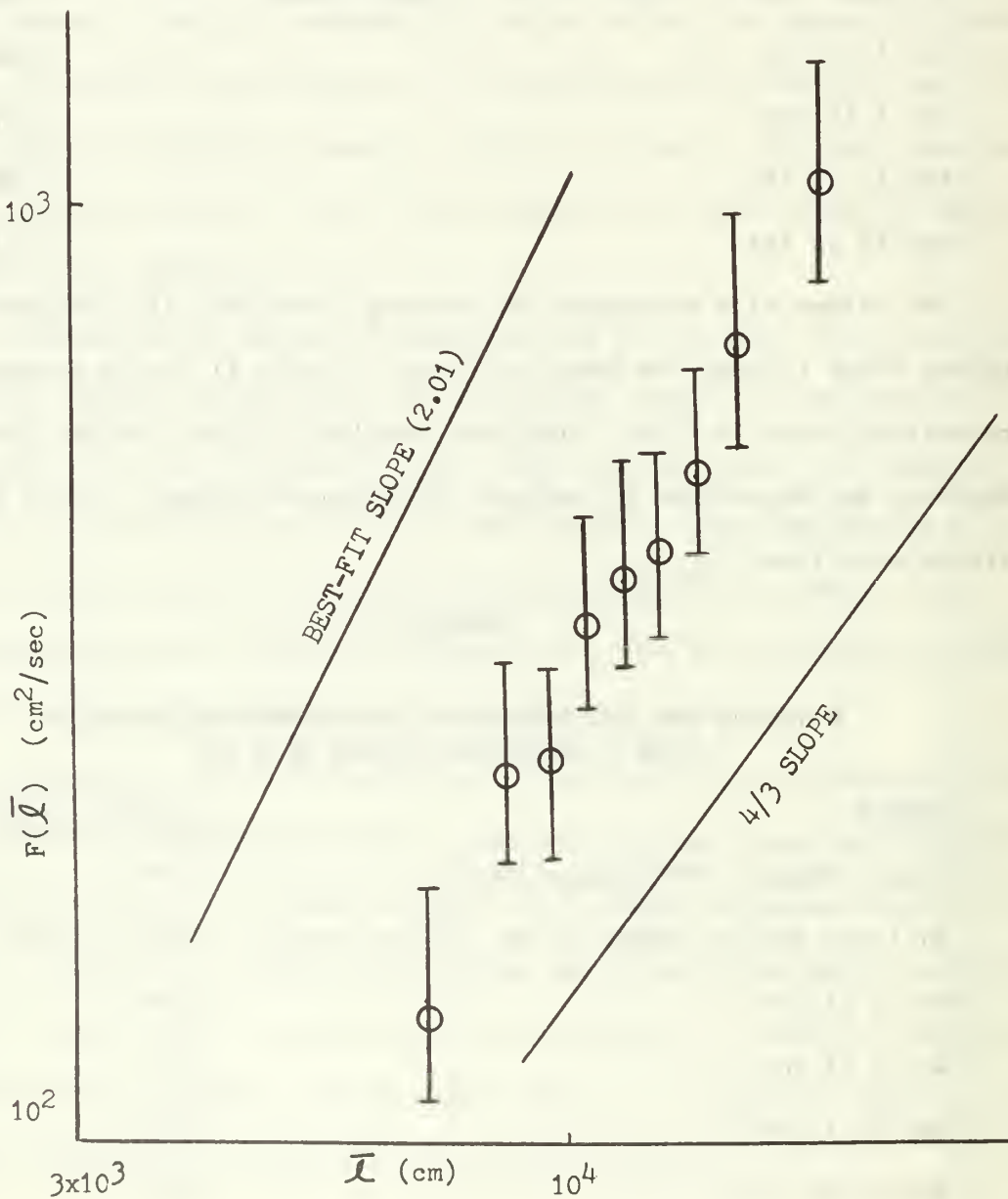


FIGURE 3
NEIGHBOR DIFFUSIVITY F
vs.
NEIGHBOR SEPARATION \bar{L}

(Run I, 13 lb. sheets, $R < 0.10$)

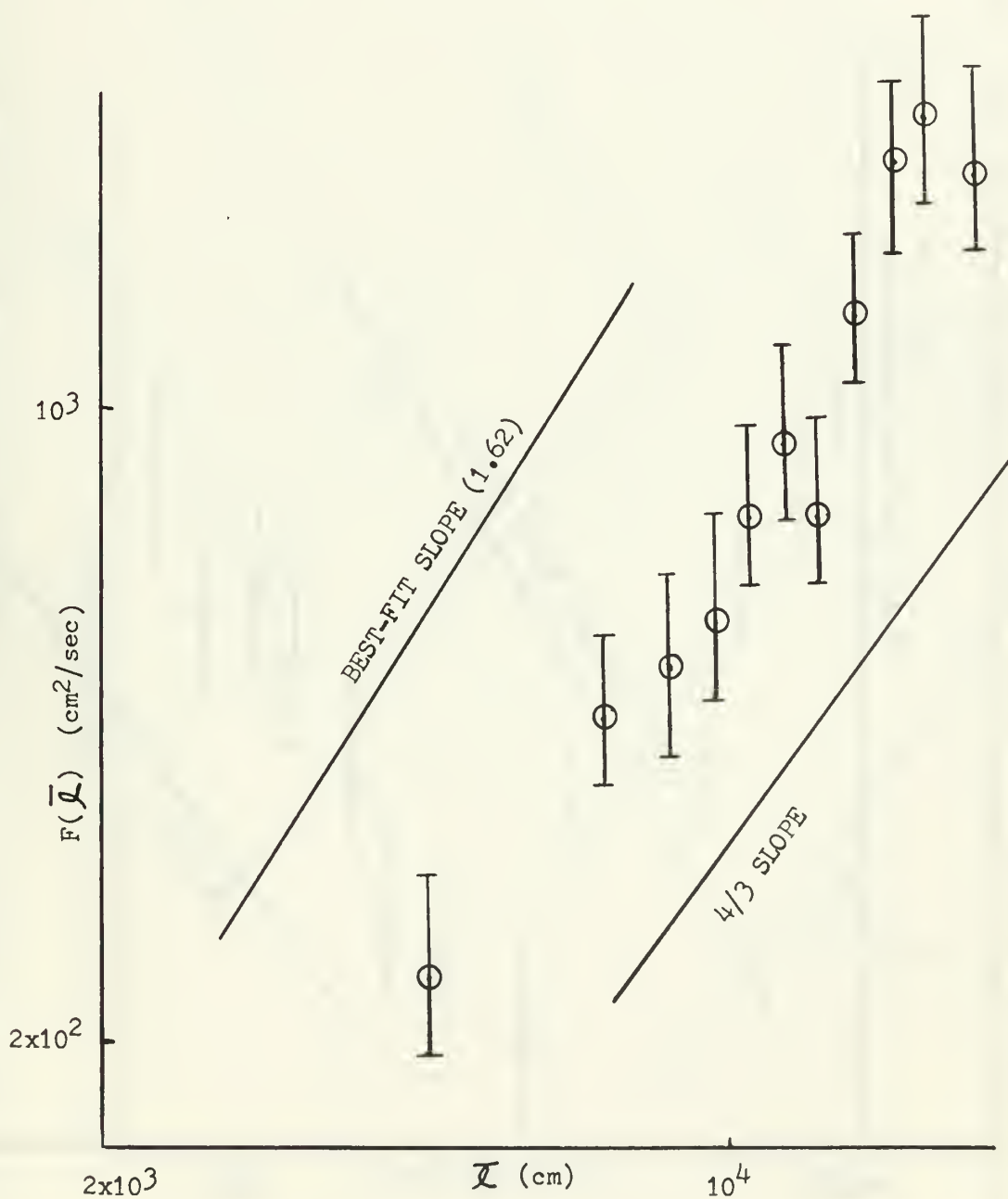


FIGURE 4
NEIGHBOR DIFFUSIVITY F
vs.
NEIGHBOR SEPARATION $\bar{\ell}$

(Run I, 15 lb. sheets, $R < 0.05$)

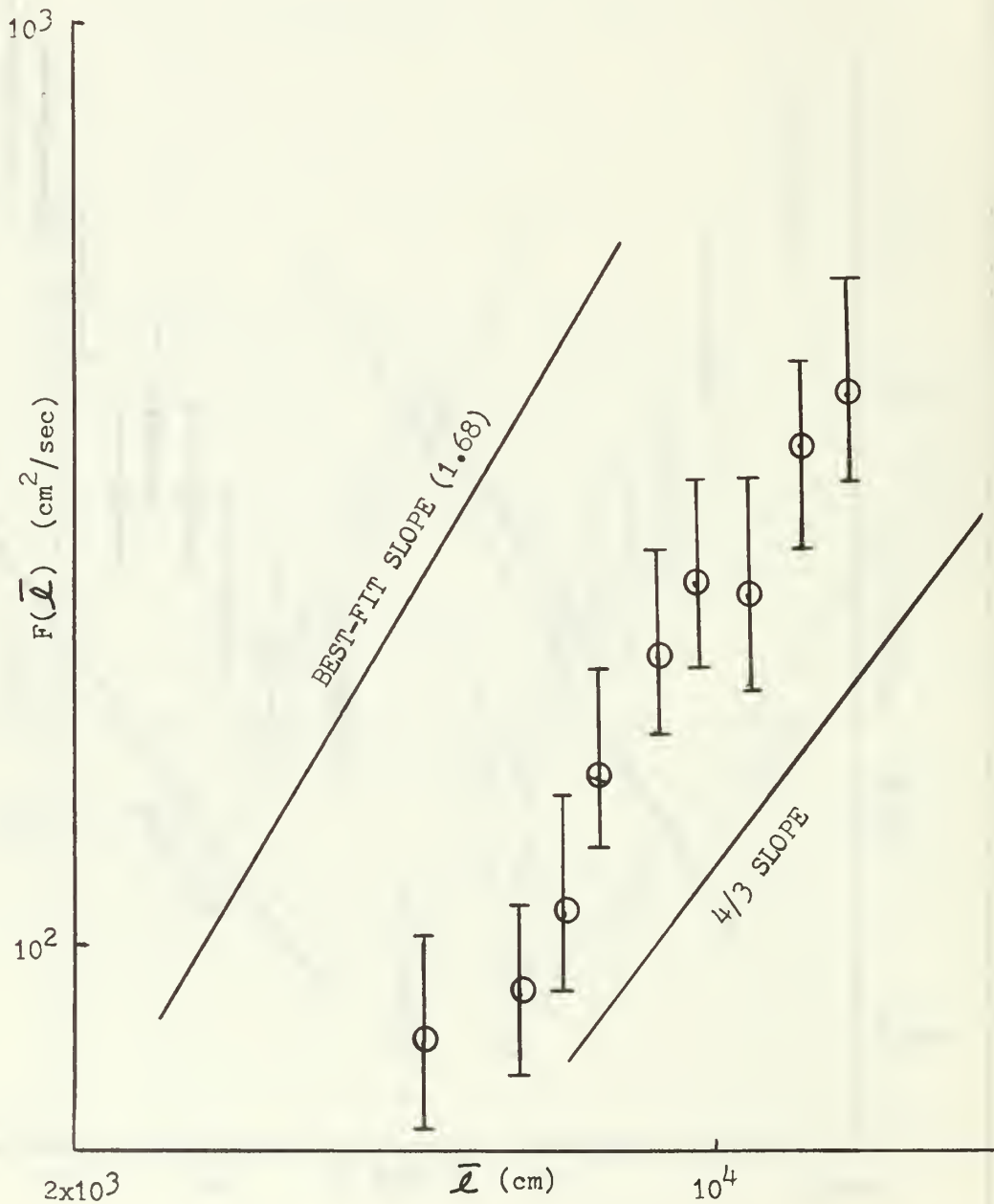


FIGURE 5
NEIGHBOR DIFFUSIVITY F
vs.
NEIGHBOR SEPARATION $\bar{\ell}$

(Run I, 15 lb. sheets, $R \leq 0.10$)

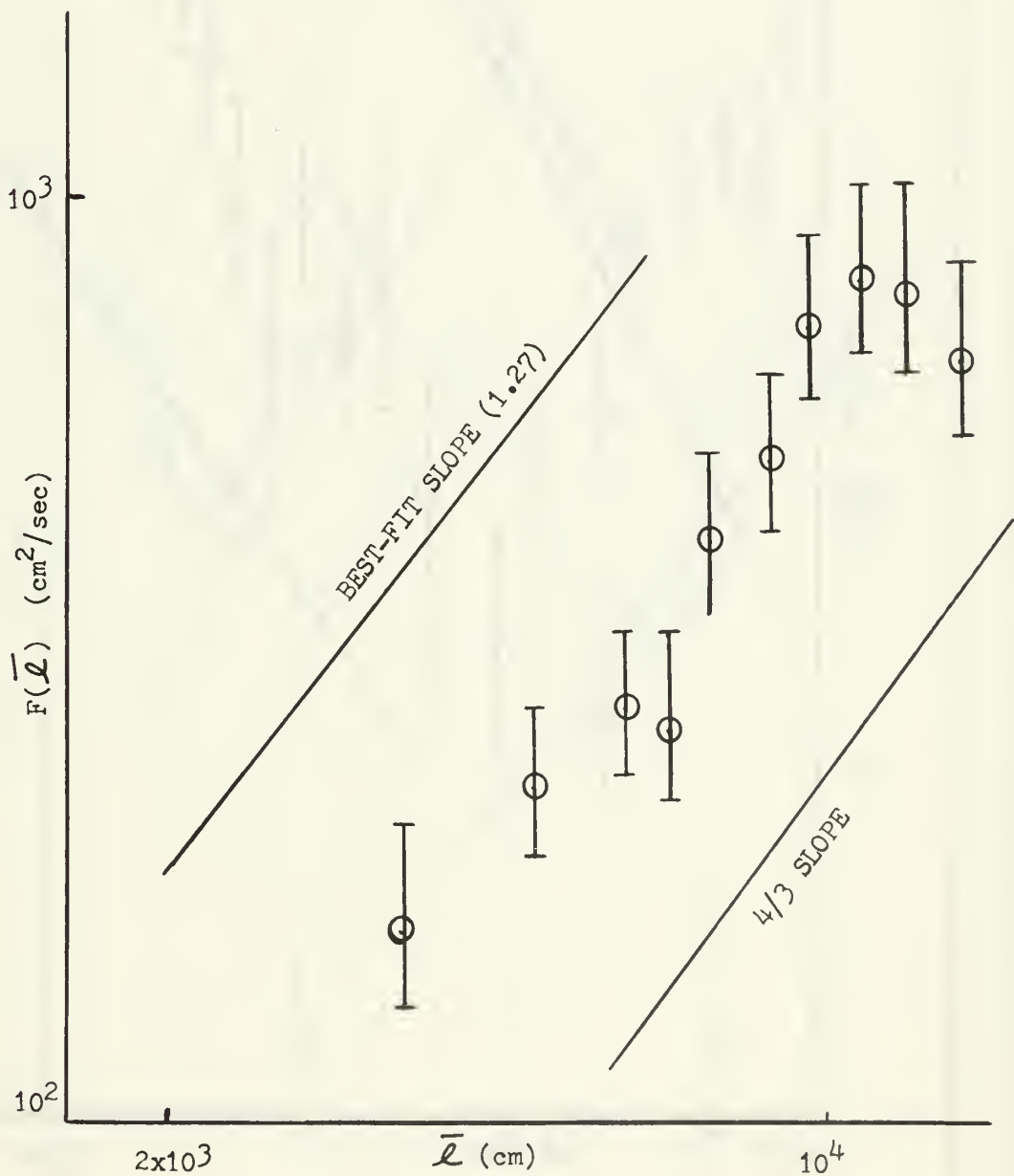


FIGURE 6
NEIGHBOR DIFFUSIVITY F
vs.
NEIGHBOR SEPARATION $\bar{\ell}$

(Run II, 13 lb. sheets, $R < 0.05$)

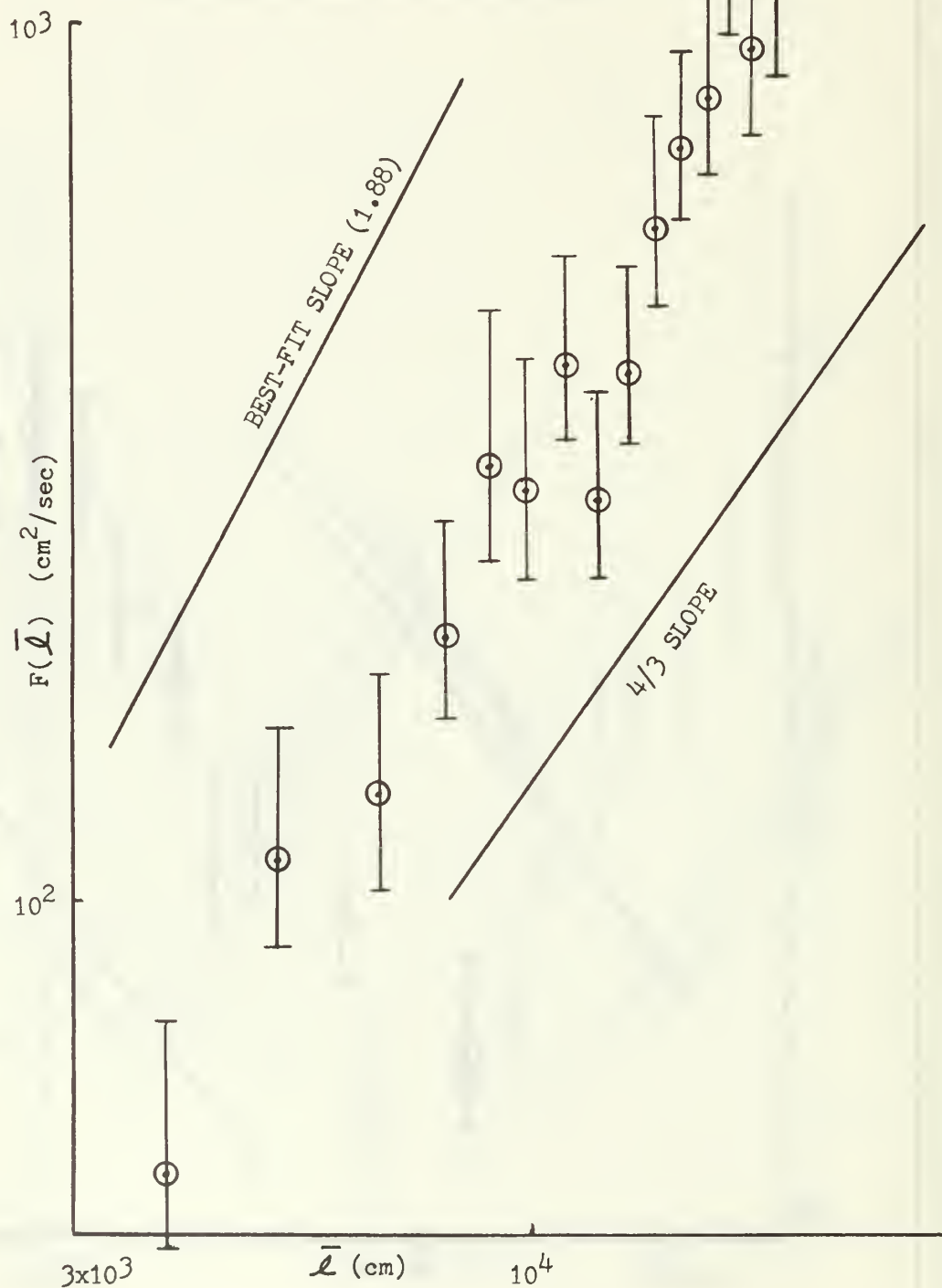


FIGURE 7
 NEIGHBOR DIFFUSIVITY F
 vs.
 NEIGHBOR SEPARATION $\bar{\ell}$
 (Run II, 13 lb. sheets, $R < 0.10$)

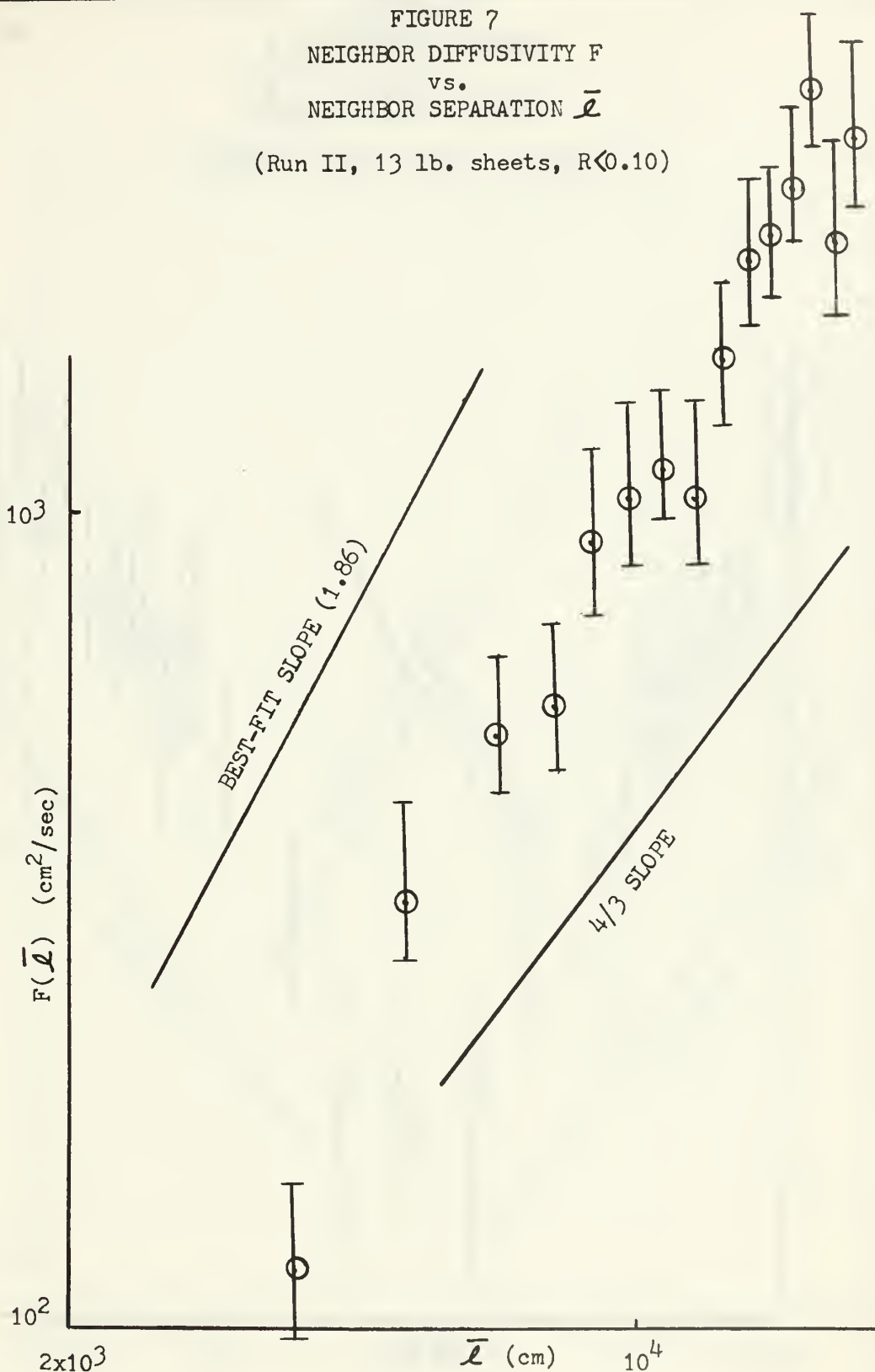


FIGURE 8
 NEIGHBOR DIFFUSIVITY F
 vs.
 NEIGHBOR SEPARATION $\bar{\ell}$

(Run II, 15 lb. sheets, $R < 0.05$)

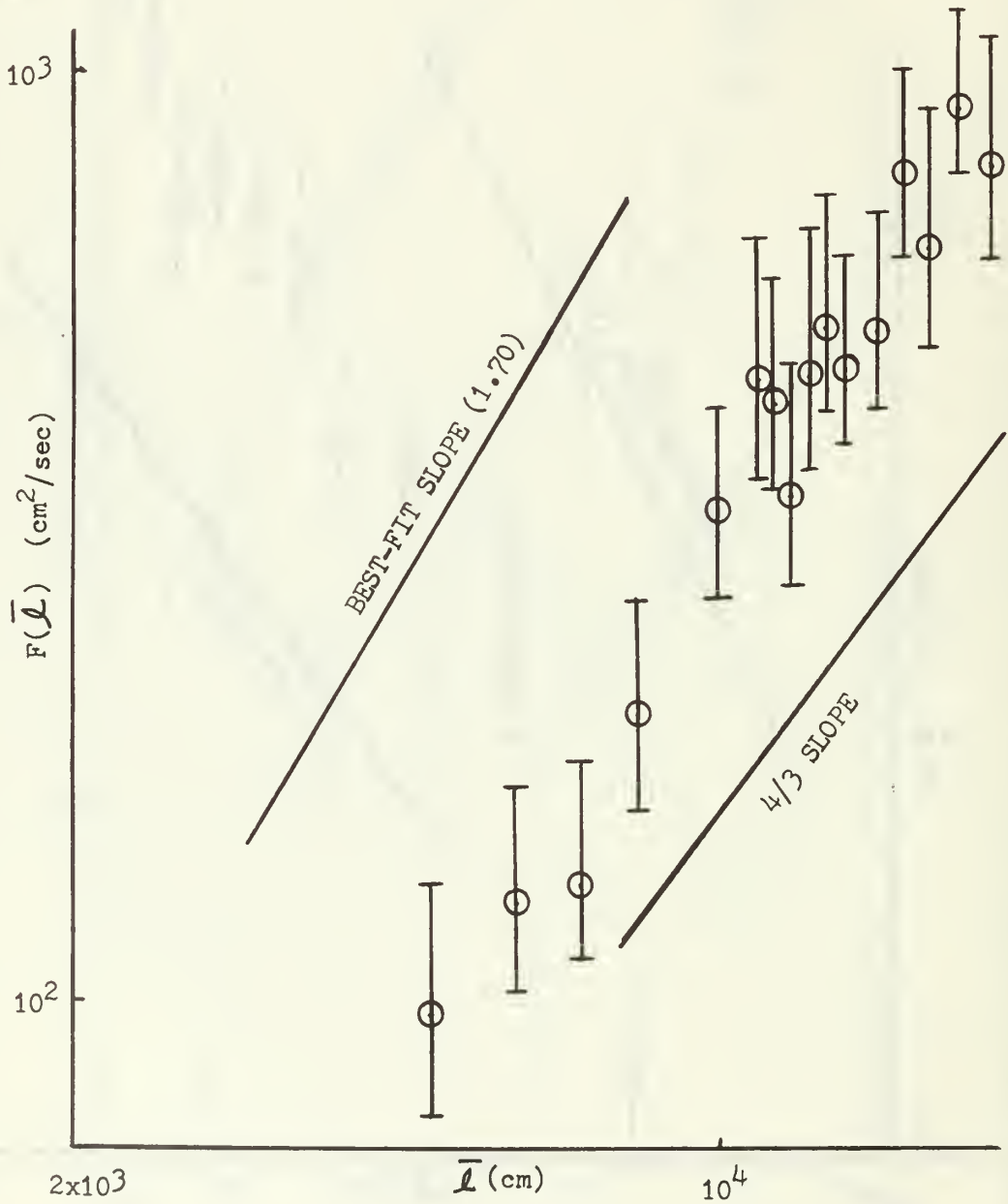


FIGURE 9
 NEIGHBOR DIFFUSIVITY F
 vs.
 NEIGHBOR SEPARATION $\bar{\ell}$

(Run II, 15 lb. sheets, $R \leq 0.10$)

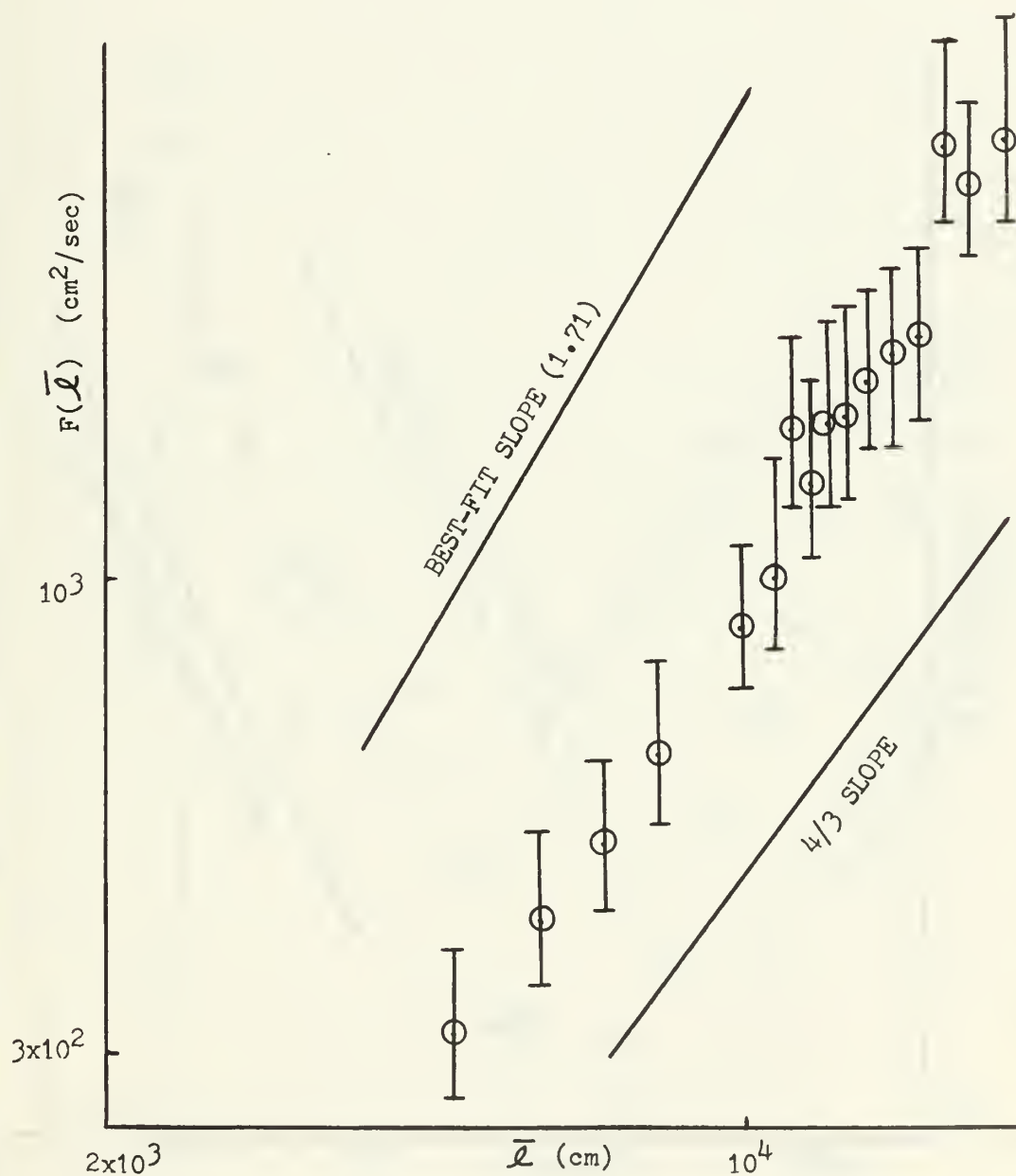


FIGURE 10
 NEIGHBOR DIFFUSIVITY F
 vs.
 NEIGHBOR SEPARATION $\bar{\ell}$

(Run I, 13 lb. sheets, $R < 0.05$) — \triangle —
 (Run II, 13 lb. sheets, $R < 0.05$) — \circ —



B. WEIGHT EFFECT

As has been pointed out by other authors, eddies with length scales less than the size of the dispersing particles cannot effect the dispersion (eg., Ozmidov, 1957). There is another side to this. Due to inertial effects, heavier particles will be less sensitive to rapid turbulent fluctuations than light particles. If the turbulent time and space scales are roughly proportional (ie., smaller eddies wiggle faster), then inertial effects will be more pronounced at smaller scales, and $F(\bar{l})$ at these scales for heavy particles should be less than $F(\bar{l})$ for light particles. As the scales increase, the effect should diminish. Thus the slope of the F, l curve should increase with weight.

The effect of size was investigated by Ozmidov (1957), who confirmed the speculation given above (although he did not mention the weight effect). A comparison between the data of Philipps, Snyder, and that analyzed here shows similar effects (Figures 12 and 13). (The slope change for Philipps is probably due mostly to weight; that for Snyder is unclear.) In an effort to isolate the weight effect, 13 lb. and 15 lb. plywood sheets of identical size and shape were used, as described above. However, the effect (although slight) was opposite to that conjectured above (Table III). This suggests that time and space scales are not proportional, and we must regard the problem of weight effect as unsettled.

C. EFFECTS OF THE REJECTION CRITERION

There has been little consideration given to the effects of varying the rejection criterion on k and on the slope of the best-fit

FIGURE 12

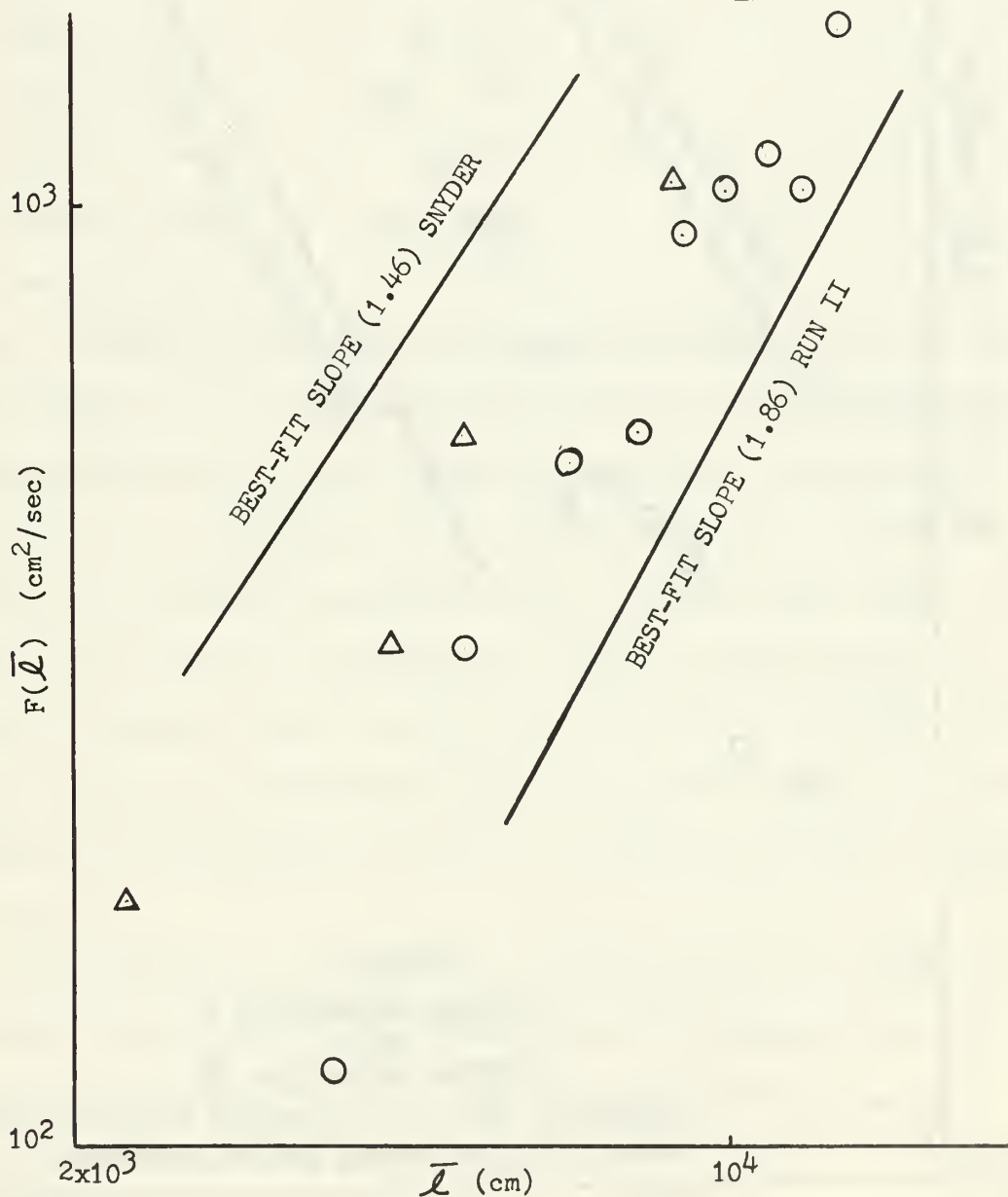
NEIGHBOR DIFFUSIVITY F

vs.

NEIGHBOR SEPARATION $\bar{\ell}$

(SNYDER, 1967, PAPER SHEETS, $R < 0.10$) — Δ —

(RUN II, 13 lb. SHEETS, $R < 0.10$) — \circ —



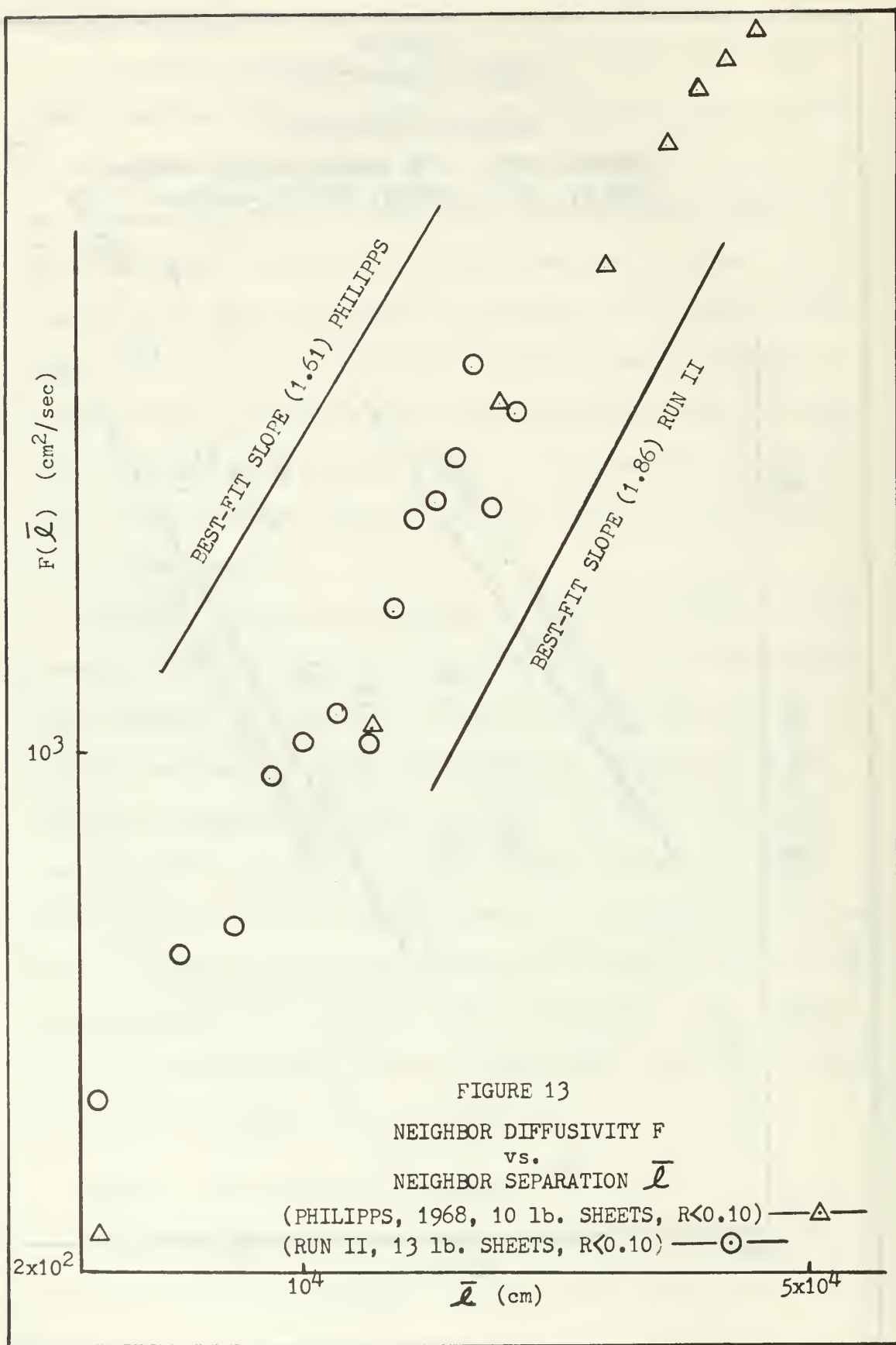


FIGURE 13

NEIGHBOR DIFFUSIVITY F
vs.
NEIGHBOR SEPARATION \bar{l}

(PHILIPPS, 1968, 10 lb. SHEETS, $R < 0.10$) — Δ —
(RUN II, 13 lb. SHEETS, $R < 0.10$) — \circ —

TABLE III

WEIGHT EFFECTS ON THE SLOPE

| <u>SOURCE</u> | <u>REJECTION FACTOR</u> | <u>SLOPE</u> |
|----------------|-------------------------|--------------|
| Run I 13 lbs. | 0.05 | 2.01 |
| Run I 15 lbs. | 0.05 | 1.68 |
| Run I 13 lbs. | 0.10 | 1.62 |
| Run I 15 lbs. | 0.10 | 1.27 |
| Run II 13 lbs. | 0.05 | 1.88 |
| Run II 15 lbs. | 0.05 | 1.70 |
| Run II 13 lbs. | 0.10 | 1.86 |
| Run II 15 lbs. | 0.10 | 1.71 |

line. It seems to be a generally accepted procedure to use the value of 0.10 for the rejection factor. To study the effects of changing the rejection factor, three values were considered: (i) $R < 0.05$, (ii) $R < 0.10$, and (iii) $R < 0.15$. It was found that only the values of 0.05 and 0.10 would be useful. For both sheets A and B the values of k were increased by a factor of approximately 2.5 when the rejection factor was increased from 0.05 to 0.10 (Table IV). Since the method of determining $F(\bar{t})$ is only valid for $R \ll 1$, this suggests that experimental values of k are high by at least a factor of two.

The variation in the slope of the best-fit line, as a result of changing the rejection factor from 0.05 to 0.10, was not as consistent as that of k . The slope decreased in Run I but remained essentially constant in Run II (Table V).

TABLE IV

EFFECTS OF THE REJECTION FACTOR ON k

| <u>SOURCE</u> | $\frac{k \text{ (cm}^{2/3} \text{ sec}^{-1})}{(R < 0.05)}$ | $\frac{k \text{ (cm}^{2/3} \text{ sec}^{-1})}{(R < 0.10)}$ |
|---------------------|--|--|
| Run I (13 lbs.) | 0.002 | 0.004 |
| Run I (15 lbs.) | 0.001 | 0.003 |
| Run II (13 lbs.) | 0.002 | 0.005 |
| Run II (15 lbs.) | 0.002 | 0.005 |

TABLE V

EFFECTS OF THE REJECTION FACTOR ON THE SLOPE

| <u>SOURCE</u> | $\frac{\text{SLOPE}}{(R < 0.05)}$ | $\frac{\text{SLOPE}}{(R < 0.10)}$ |
|---------------------|-----------------------------------|-----------------------------------|
| Run I (13 lbs.) | 2.01 | 1.62 |
| Run I (15 lbs.) | 1.68 | 1.27 |
| Run II (13 lbs.) | 1.88 | 1.86 |
| Run II (15 lbs.) | 1.70 | 1.71 |

D. MEAN SEPARATION

It was noted in the initial results that the separation between diffusers was not continuously increasing as was expected. Therefore, the mean separations of all pairs of diffusers were calculated for each pass of Runs I and II. The results of the mean separation

versus total time are shown in Figures 14 and 15. The variation of the mean separation with time suggested the possibility of wave action affecting the diffusion. To study the effects of wave action the relationship between $F(l)$ and wave spectra was determined.

E. EFFECTS OF WAVE ACTION

In this study let $F(l) \approx \frac{\overline{(\Delta l)^2}}{2\Delta t}$ where l is the separation between two particles, and the bar denotes an ensemble average. If the sea surface is disturbed by a monochromatic wave, then $\eta(x) = a \sin(\kappa x - \omega t)$ where η is the free surface height, a is wave amplitude, κ is wave number, x is horizontal distance, ω is angular frequency, and t is time. Consider two particles located on the x -axis and separated by a mean distance l . Then $l_0 = l + \epsilon_0$ and $l_1 = l + \epsilon_1$, where ϵ is the change of separation due to wave motion. Since sampling time (minutes) is much larger than wave period (sec), ϵ_0 and ϵ_1 are independent.

$$\begin{aligned} \text{Thus, } \overline{(\Delta l)^2} &= \overline{(l_1 - l_0)^2} = \overline{(\epsilon_1 - \epsilon_0)^2} \\ &= \overline{\epsilon_1^2} + \overline{\epsilon_0^2} - 2\overline{\epsilon_1 \epsilon_0} \\ &= \overline{\epsilon_1^2} + \overline{\epsilon_0^2}. \end{aligned}$$

Thus, $\overline{(\Delta l)^2} = 2\overline{\epsilon_1^2} = 2\overline{\epsilon^2}$ since the mean quantities are independent of time. Here, the bar denotes either a time or an ensemble average.

The value of ϵ must now be calculated. Assume the particles to be located at mean positions 0 and l . As both particles move in a circular orbit of radius a ,

$$\epsilon(l, t) = a \sin(-\omega t) - a \sin(\kappa l - \omega t)$$

and

$$\frac{\epsilon^2}{a^2} = \sin^2 \omega t + \sin^2(\kappa l - \omega t) + 2 \sin \omega t \sin(\kappa l - \omega t).$$

Averaging:

$$\overline{\frac{\epsilon^2(t)}{a^2}} = 1 + 2 \sin \omega t \sin (\kappa l - \omega t).$$

Simplifying the equation using trigonometric identities gives

$$\overline{\frac{\epsilon^2(t)}{a^2}} = 1 - \cos \kappa l. \quad \text{Consider } l \text{ as some fraction (N) of the wavelength } (\lambda): l = N\lambda. \quad \text{The equation now takes the form}$$

$$\overline{\frac{\epsilon^2(N)}{a^2}} = 1 - \cos 2\pi N \quad (1)$$

since $\lambda = \frac{2\pi}{\kappa}$. Thus,

$$F_w \approx \overline{\frac{\epsilon^2}{\Delta t}} = \frac{a^2}{\Delta t} (1 - \cos 2\pi N). \quad (2)$$

Suppose that N is small. This is a reasonable assumption since a 16-second swell has a wavelength of about 1200 feet. Then if l is approximately 200 feet, $N \approx 1/6$, and $2\pi N \approx 1$. Since $\cos 1 = 0.54$, and the expansion $\cos x \approx 1 - x^2/2$ gives 0.5, we can use the expansion in (2), giving

$$F_w \approx \frac{2\pi^2 a^2}{\Delta t} N^2, \quad N \leq 1/6, \quad (3)$$

which implies an N^2 relationship (ie. $F \propto l^2$). The power will decrease as N becomes larger (so that the approximation is no longer valid).

The direction of the line between two particles must be considered. Normally this line will be at some angle (θ) to the direction of wave travel. Then,

$$\epsilon(l, t) = \left\{ a \sin(-\omega t) - a \sin(\kappa l - \omega t) \right\} \cos \theta.$$

When averaging over the direction, $\overline{\cos^2 \theta} = 1/2$. Thus, (2) becomes

$$F_w \approx \frac{a^2}{2\Delta t} (1 - \cos 2\pi N), \quad (4)$$

and (3) becomes

$$F_w \approx \frac{\pi a^2}{\Delta t} N^2, \quad N \leq 1/6. \quad (5)$$

Consider two wave trains at angles θ_1 and θ_2 to the line between particles. Then,

$$\epsilon(t, t) = a_1 \left\{ \omega_1, n_1 \right\} \cos \theta_1 + a_2 \left\{ \omega_2, n_2 \right\} \cos \theta_2.$$

Since the waves are independent,

$$\overline{\epsilon^2} = a_1^2 (1 - \cos 2\pi N_1) + a_2^2 (1 - \cos 2\pi N_2). \quad (6)$$

Equation (6) can be generalized to any number of wave trains, M. Then using equation (5) gives

$$F_w(t) = \frac{\pi}{\Delta t} \sum a_i^2 N_i^2$$

or

$$F_w(t) = \frac{t^2}{4\Delta t} \sum n_i^2 a_i^2, \quad t \leq \frac{\lambda}{6}. \quad (7)$$

In terms of energy spectrum $S(n)$, (7) becomes $F_w(t) = \frac{t^2}{4\Delta t} \int n^2 S(n) dn$, where the integral is the mean square surface slope. From (4), the exact form of (7) is

$$F_w(t) = \frac{1}{2\Delta t} \sum a_i^2 (1 - \cos n_i t). \quad (8)$$

Then the neighbor diffusivity due to wave motion can be calculated from a measured energy spectrum.

The wave records at a position close to that where the experiments were conducted were available for the days the data were collected. A power spectrum was determined for each day using a digital computer (Figure 16). With the power spectra $F_w(t)$ could be calculated using various values of t (Appendix C). The power spectrum for Run II was chosen since it contained more energy than that for Run I. From

Figure 17, which represents the data for Run II in Appendix C, it was determined that the wave action had little direct effect on the diffusion of the particles.

FIGURE 14
MEAN SEPARATION
vs.
TOTAL TIME

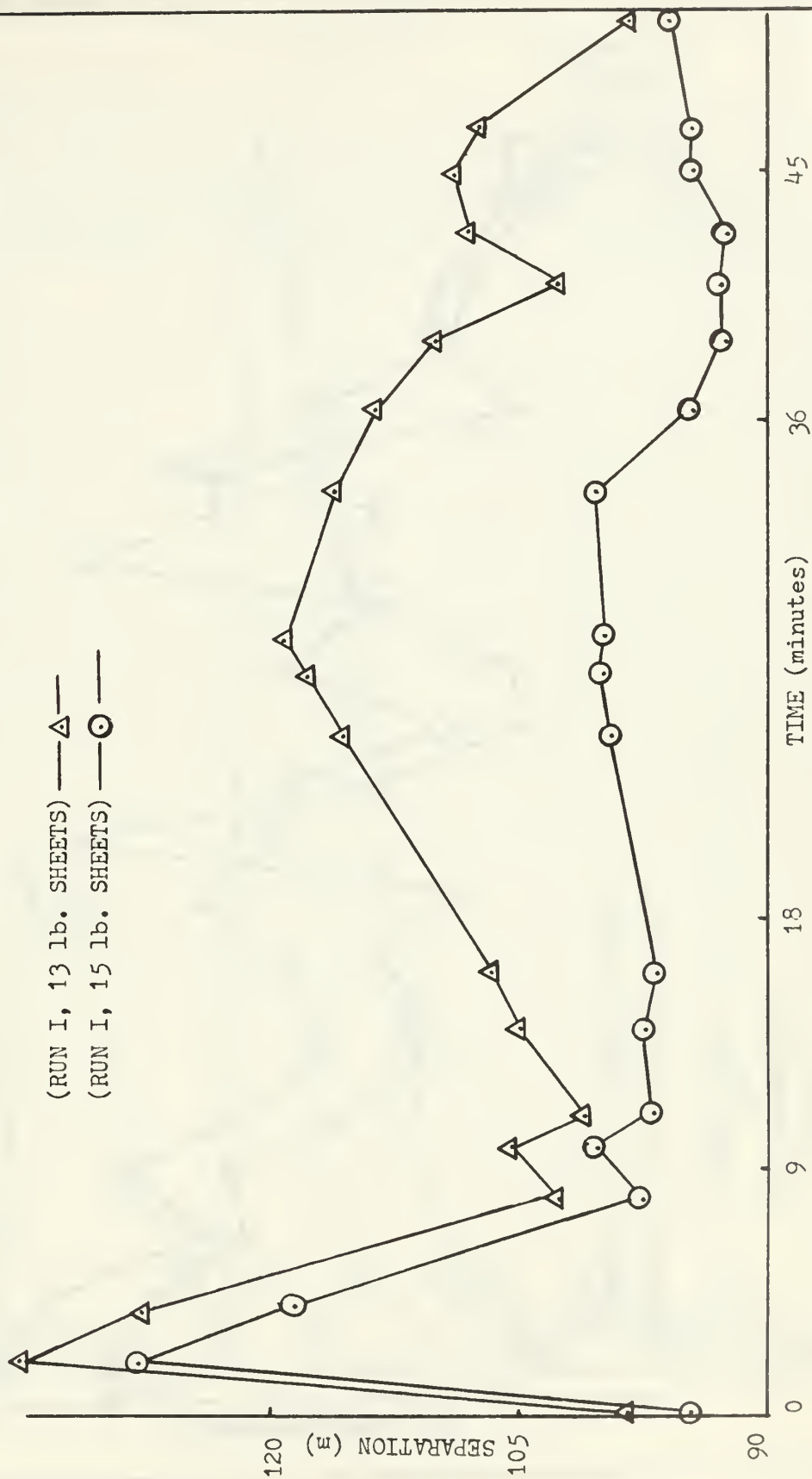


FIGURE 15
MEAN SEPARATION
vs.
TOTAL TIME

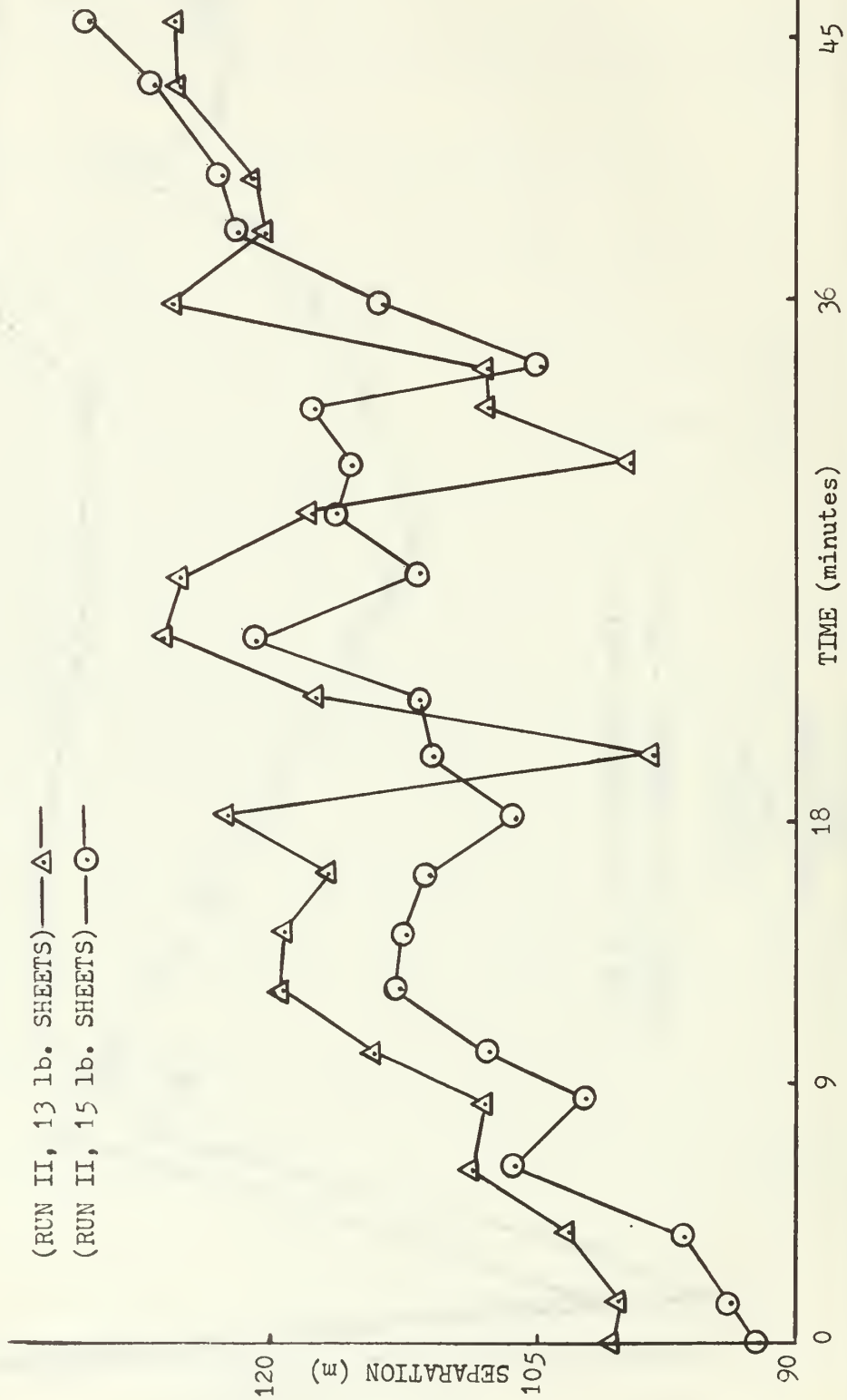
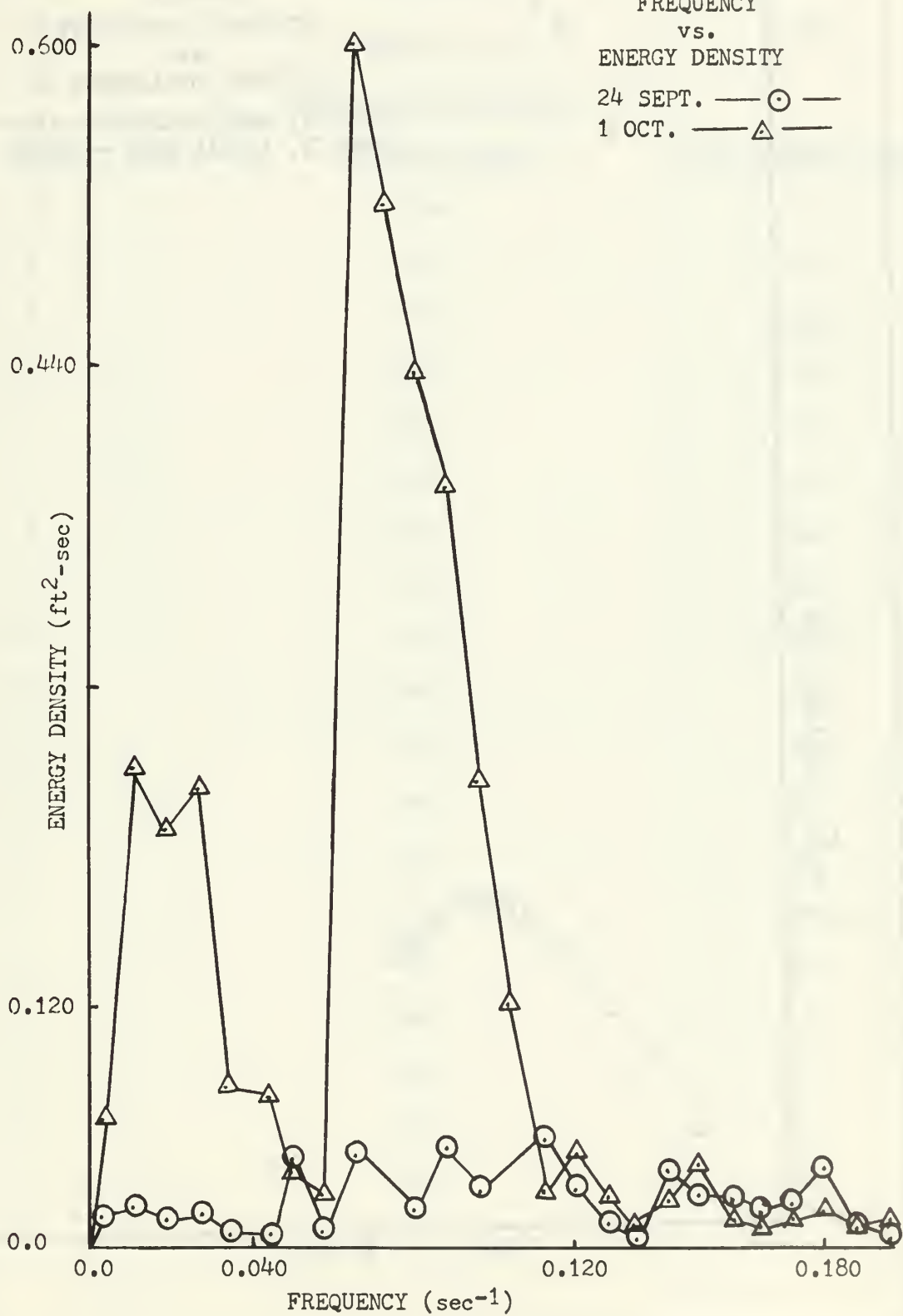
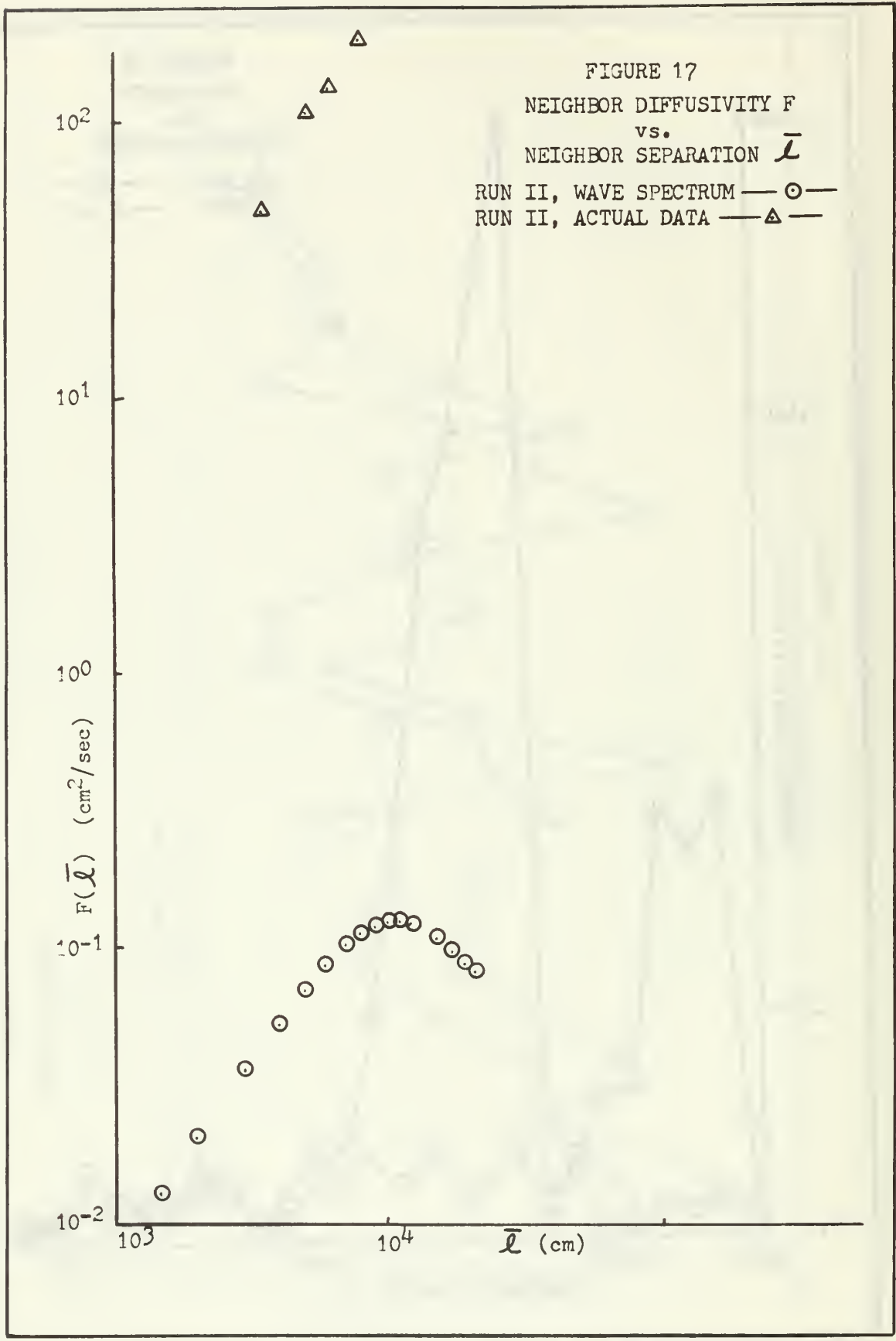


FIGURE 16
 FREQUENCY
 vs.
 ENERGY DENSITY
 24 SEPT. —○—
 1 OCT. —△—





APPENDIX A

FLIGHT DATA

Flight Data for Run I

| <u>PASS</u> | <u>ALTITUDE (FEET)</u> | <u>TIME INTERVAL (SEC)</u> |
|-------------|------------------------|----------------------------|
| 1 | 750 | --- |
| 2 | 875 | 110 |
| 3 | 965 | 112 |
| 5 | 1,100 | 250 |
| 6 | 1,090 | 112 |
| 7 | 1,180 | 117 |
| 8 | 1,235 | 126 |
| 9 | 920 | 130 |
| 10 | 940 | 511 |
| 11 | 1,045 | 133 |
| 12 | 995 | 136 |
| 14 | 895 | 284 |
| 15 | 930 | 165 |
| 16 | 1,000 | 138 |
| 17 | 995 | 122 |
| 18 | 1,000 | 114 |
| 19 | 900 | 127 |
| 20 | 955 | 110 |
| 22 | 960 | 259 |
| 23 | 975 | 131 |

Flight Data for Run II

| <u>PASS</u> | <u>ALTITUDE (FEET)</u> | <u>TIME INTERVAL (SEC)</u> |
|-------------|------------------------|----------------------------|
| 1 | 865 | --- |
| 2 | 815 | 110 |
| 3 | 925 | 133 |
| 4 | 865 | 122 |
| 5 | 955 | 113 |
| 6 | 1,060 | 128 |
| 7 | 1,040 | 117 |
| 8 | 900 | 122 |
| 9 | 1,000 | 118 |
| 10 | 940 | 119 |
| 11 | 895 | 124 |
| 12 | 860 | 132 |
| 13 | 1,045 | 113 |
| 14 | 920 | 125 |
| 15 | 930 | 130 |
| 16 | 950 | 105 |
| 17 | 855 | 110 |
| 18 | 860 | 98 |
| 19 | 940 | 129 |
| 20 | 940 | 128 |
| 21 | 1,015 | 125 |
| 22 | 1,010 | 135 |
| 23 | 875 | 124 |

APPENDIX B

RESULTS OF ANALYSIS

Run I, 13 Pound Diffusers, R < 0.05

| <u>CLASS INTERVALS (m)</u> | <u>NUMBER OF VALUES</u> | <u>RESULTS (m²/sec)</u> |
|----------------------------|-------------------------|------------------------------------|
| 65-80 | 56 | F(71.31) = 0.01356 |
| 80-90 | 57 | F(85.96) = 0.02444 |
| 90-100 | 44 | F(95.67) = 0.02500 |
| 100-110 | 63 | F(104.92) = 0.03522 |
| 110-120 | 52 | F(114.55) = 0.03931 |
| 120-130 | 61 | F(125.43) = 0.04186 |
| 130-145 | 64 | F(137.53) = 0.05098 |
| 145-160 | 45 | F(151.82) = 0.06979 |
| 180-195 | 46 | F(186.49) = 0.10282 |

Run I, 13 Pound Diffusers, R < 0.10

| | | |
|---------|----|---------------------|
| 40-55 | 56 | F(45.87) = 0.02329 |
| 65-80 | 85 | F(71.87) = 0.04522 |
| 80-90 | 75 | F(85.81) = 0.05129 |
| 90-100 | 53 | F(95.60) = 0.05807 |
| 100-110 | 81 | F(105.07) = 0.07598 |
| 110-120 | 69 | F(114.68) = 0.09095 |
| 120-130 | 70 | F(125.35) = 0.07648 |
| 130-145 | 90 | F(137.51) = 0.12774 |
| 145-160 | 62 | F(152.11) = 0.18212 |
| 160-175 | 50 | F(165.58) = 0.21077 |
| 180-195 | 56 | F(187.00) = 0.18107 |

Run I, 15 Pound Diffusers, R < 0.05

| <u>CLASS INTERVAL (m)</u> | <u>NUMBER OF VALUES</u> | <u>RESULTS (m²/sec)</u> |
|---------------------------|-------------------------|------------------------------------|
| 40-55 | 59 | F(47.90) = 0.00779 |
| 55-65 | 64 | F(60.52) = 0.00884 |
| 65-70 | 56 | F(67.58) = 0.01086 |
| 70-80 | 62 | F(74.42) = 0.01543 |
| 80-90 | 55 | F(85.84) = 0.02056 |
| 90-100 | 53 | F(94.38) = 0.02477 |
| 100-115 | 50 | F(107.58) = 0.02384 |
| 115-130 | 47 | F(122.59) = 0.03485 |
| 130-145 | 56 | F(136.97) = 0.03984 |

Run I, 15 Pound Diffusers, R < 0.10

| | | |
|---------|----|---------------------|
| 25-40 | 65 | F(34.22) = 0.01626 |
| 40-55 | 98 | F(47.90) = 0.02275 |
| 55-65 | 95 | F(60.27) = 0.02819 |
| 65-70 | 69 | F(67.59) = 0.02667 |
| 70-80 | 93 | F(74.31) = 0.04279 |
| 80-90 | 79 | F(85.80) = 0.05201 |
| 90-100 | 75 | F(94.47) = 0.07250 |
| 100-115 | 69 | F(107.43) = 0.08183 |
| 115-130 | 55 | F(122.54) = 0.07943 |
| 130-145 | 64 | F(137.01) = 0.06640 |

Run II, 13 Pound Diffusers, R < 0.05

| <u>CLASS INTERVAL (m)</u> | <u>NUMBER OF VALUES</u> | <u>RESULTS (m²/sec)</u> |
|---------------------------|-------------------------|------------------------------------|
| 30-45 | 39 | F(37.51) = 0.00488 |
| 45-60 | 42 | F(51.67) = 0.01120 |
| 60-75 | 45 | F(67.50) = 0.01312 |
| 75-85 | 51 | F(80.33) = 0.01982 |
| 85-95 | 33 | F(90.09) = 0.03118 |
| 95-105 | 43 | F(99.19) = 0.02912 |
| 105-115 | 59 | F(109.85) = 0.04057 |
| 115-125 | 53 | F(120.11) = 0.02847 |
| 125-135 | 61 | F(130.36) = 0.03986 |
| 135-145 | 55 | F(139.87) = 0.05761 |
| 145-155 | 67 | F(150.34) = 0.07082 |
| 155-165 | 58 | F(160.84) = 0.08120 |
| 165-175 | 40 | F(169.51) = 0.12072 |
| 175-185 | 43 | F(180.41) = 0.09218 |
| 185-200 | 41 | F(191.28) = 0.10879 |

Run II, 13 Pound Diffusers, R < 0.10

| | | |
|---------|----|---------------------|
| 30-45 | 81 | F(37.62) = 0.01185 |
| 45-60 | 72 | F(52.10) = 0.03320 |
| 60-75 | 83 | F(67.81) = 0.05298 |
| 75-85 | 78 | F(80.33) = 0.05729 |
| 86-95 | 55 | F(89.86) = 0.09132 |
| 95-105 | 70 | F(99.69) = 0.10325 |
| 105-115 | 91 | F(109.64) = 0.11272 |
| 115-125 | 74 | F(120.06) = 0.10330 |
| 125-135 | 95 | F(130.23) = 0.15158 |

| <u>CLASS INTERVAL (m)</u> | <u>NUMBER OF VALUES</u> | <u>RESULTS (m²/sec)</u> |
|---------------------------|-------------------------|------------------------------------|
| 135-145 | 88 | F(140.08) = 0.20152 |
| 145-155 | 97 | F(150.09) = 0.21462 |
| 155-165 | 91 | F(160.26) = 0.24438 |
| 165-175 | 64 | F(169.53) = 0.32294 |
| 175-185 | 54 | F(180.44) = 0.20824 |
| 185-200 | 58 | F(191.42) = 0.28200 |

Run II, 15 Pound Diffusers, R < 0.05

| | | |
|---------|----|---------------------|
| 40-55 | 45 | F(48.54) = 0.00967 |
| 55-65 | 52 | F(59.90) = 0.01271 |
| 65-75 | 59 | F(70.01) = 0.01323 |
| 75-90 | 53 | F(80.81) = 0.02023 |
| 90-105 | 60 | F(98.94) = 0.03340 |
| 105-110 | 39 | F(107.53) = 0.04644 |
| 110-115 | 48 | F(112.29) = 0.04404 |
| 115-120 | 45 | F(117.78) = 0.03473 |
| 120-125 | 38 | F(122.73) = 0.04667 |
| 125-130 | 50 | F(127.10) = 0.05341 |
| 130-140 | 59 | F(134.89) = 0.04745 |
| 140-150 | 55 | F(144.84) = 0.05272 |
| 150-160 | 62 | F(155.33) = 0.07673 |
| 160-170 | 39 | F(164.89) = 0.06392 |
| 170-185 | 57 | F(177.51) = 0.09138 |
| 185-200 | 38 | F(192.15) = 0.07876 |

Run II, 15 Pound Diffusers, R < 0.10

| <u>CLASS INTERVAL (m)</u> | <u>NUMBER OF VALUES</u> | <u>RESULTS (m²/sec)</u> |
|---------------------------|-------------------------|------------------------------------|
| 40-55 | 87 | F(48.43) = 0.03174 |
| 55-65 | 87 | F(59.65) = 0.04248 |
| 65-75 | 92 | F(70.14) = 0.05138 |
| 75-90 | 80 | F(80.60) = 0.06454 |
| 90-105 | 89 | F(98.57) = 0.08951 |
| 105-110 | 57 | F(107.58) = 0.10190 |
| 110-115 | 79 | F(112.48) = 0.14685 |
| 115-120 | 67 | F(117.65) = 0.12690 |
| 120-125 | 64 | F(122.67) = 0.14780 |
| 125-130 | 70 | F(127.19) = 0.15046 |
| 130-140 | 85 | F(135.11) = 0.16463 |
| 140-150 | 79 | F(145.11) = 0.17404 |
| 150-160 | 86 | F(155.20) = 0.18276 |
| 160-170 | 64 | F(164.92) = 0.29894 |
| 170-185 | 83 | F(176.35) = 0.26843 |
| 185-200 | 51 | F(192.13) = 0.30326 |

APPENDIX C

RUN II WAVE SPECTRUM RESULTS

| <u>l (m)</u> | <u>F(l) (m²/sec)</u> (x 10 ⁻⁴) |
|--------------|--|
| 10.0 | 0.00617 |
| 15.0 | 0.01290 |
| 20.0 | 0.02085 |
| 30.0 | 0.03741 |
| 40.0 | 0.05350 |
| 50.0 | 0.07032 |
| 60.0 | 0.08748 |
| 70.0 | 0.10236 |
| 80.0 | 0.11364 |
| 90.0 | 0.12175 |
| 100.0 | 0.12645 |
| 110.0 | 0.12682 |
| 120.0 | 0.12356 |
| 150.0 | 0.10919 |
| 170.0 | 0.09773 |
| 190.0 | 0.08814 |
| 210.0 | 0.08369 |

BIBLIOGRAPHY

- Batchelor, G. K., "Kolmogoroff's Theory of Locally Isotropic Turbulence," Proc. Camb. Phil. Soc., v. 43, p. 533-559, December 1946.
- Batchelor, G. K., "The Application of the Similarity Theory of Turbulence to Atmospheric Diffusion," Quart. Jour. Roy. Met. Soc., v. 76, p. 133-146, October 1949.
- Champion, R. H., A Field Study of Oceanic Turbulent Horizontal Diffusion, M.S. Thesis, Naval Postgraduate School, Monterey, December 1968.
- Denner, W. W., Green, T., and Snyder, W. H., "Large Scale Oceanic Droque Diffusion," Nature, v. 219, no. 5152, p. 361-362, July 1968.
- Gray, E., and Pochapsky, T. E., "Surface Dispersion Experiments and Richardson's Diffusion Equation," Jour. Geophy. Research, v. 69, no. 24, p. 5155-5159, December 1964.
- Osmidov, R. V., "Experimental Investigation of Horizontal Turbulent Diffusion in the Sea and in a Shallow Artificial Reservoir," Investia Akad Nauk SSSR, geofiz, v. 6, p. 756-764, 1957.
- Philipps, G., A Field Study of Oceanic Turbulent Horizontal Diffusion, M.S. Thesis, Naval Postgraduate School, Monterey, June 1968.
- Richardson, L. F., "Atmospheric Diffusion Shown on a Distance-Neighbour Graph," Proc. Roy. Soc. London, v. 110A, p. 709-737, 1926.
- Snyder, W. H., A Field Test of the Four-Thirds Law of Horizontal Diffusion in the Ocean, M.S. Thesis, Naval Postgraduate School, Monterey, June 1967.
- Stommel, H., "Horizontal Diffusion Due to Oceanic Turbulence," Jour. Marine Research, v. 8, no. 3, p. 199-225, 1949.

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| | | 2b. GROUP | |
| 3. REPORT TITLE Neighbor Diffusion in Monterey Bay | | | |
| 4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Master's Thesis; October 1969 | | | |
| 5. AUTHOR(S) (First name, middle initial, last name) "B""J" Taylor, Jr. | | | |
| 6. REPORT DATE October 1969 | | 7a. TOTAL NO. OF PAGES 53 | 7b. NO. OF REFS 10 |
| 8a. CONTRACT OR GRANT NO. | | 9a. ORIGINATOR'S REPORT NUMBER(S) | |
| b. PROJECT NO. | | | |
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| 14 | KEY WORDS | LINK A | | LINK B | | LINK C | |
|----|-----------|--------|----|--------|----|--------|----|
| | | ROLE | WT | ROLE | WT | ROLE | WT |
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